PROBLEM SET 4

It's OK to co-operate with classmates on problem sets. If you get stuck on a problem, don't waste a lot of time on it --- you have better things to do.

The following problems from Starr's *General Equilibrium Theory*, *2nd edition*, are assigned.

21.1

22.1

22.2

22.3

In addition, one problem adapted from the June 2010 qual is assigned, attached below.

4. Consider core convergence in a pure exchange economy becoming large through Q-fold replication.

4.1. Consider an example where there are two commodities, x and y, and two trader types, 1 and 2.

Type 1 is characterized as having utility function

$$\begin{aligned} u^1(x,y) &= xy, \text{and endowment} \\ r^1 &= (99,1). \end{aligned}$$

Type 2 is characterized as having utility function

$$\begin{aligned} u^2(x,y) &= xy, \text{and endowment} \\ r^2 &= (1,99). \end{aligned}$$

Show that the following allocation, a^1 to type 1 and a^2 to type 2, is in the core for the original economy with one of each type, and is not in the core for an economy with $Q \geq 2$: $a^1 = (90,90); a^2 = (10,10)$.

4.2. H represents an economy with a finite number of households of strictly convex, continuous preferences; the typical endowment is r^h and the typical allocation is x^h for $h \in H$ with preferences $\succeq_h$. Let Q be a positive integer. Let Core $(Q \times H)$ denote the set of core allocations of the Q-fold replica of the original economy H. Under the equal-treatment property, a typical core allocation will be represented by allocations to type, $\{x^h | h \in H\}$. Recall that blocking coalitions do not need to provide equal treatment in the blocking allocation. Denote the set of households of this economy as $Q \times H = \{h,q | h \in H, q = 1,2,...,Q\}$, where "$h,q$" is read as "the qth household of type h."

Demonstrate that Core $((Q+1) \times H) \subseteq$ Core$(Q \times H)$.

The question ends here. However, if your memory of core convergence is a bit thin you may find the following definitions useful:

A **coalition** is any subset $S \subseteq H$.

An allocation $\{x^i, i{\in}H\}$ is **blocked** by a coalition $S{\subseteq}H$ if there is an assignment $\{y^i, i \in S\}$ so that:

1. $\sum_{i \in S} y^i \leq \sum_{i \in S} r^i$ (where the inequality holds coordinatewise),
2. $y^i {\succeq_i} x^i$, for all $i \in S$, and
3. $y^{i'} {\succ_{i'}} x^{i'}$, for some $i' \in S$

The **core** of the economy is the set of feasible allocations that are not blocked by any coalition $S{\subseteq}H$.